TIME IS MONEY: PART 2

By

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In Part 1 of "The Time Value of Money" we examined the fundamental mathematics of interest calculations and their impact on the lives of professionals and business people. In this part, we explore these fundamental principles, which affect the determination of future and present values, and which we use in retirement and tax planning and in fund management.

Future Value

Future value is defined as the sum to which an amount, or a series of periodic equal amounts, will grow at the end of a period of time, invested at a particular *compound* interest rate. The future value of a principal sum of money depends upon the rate of interest, the mode of calculation of the interest, and the duration of investment.

As noted above, there are two forms of interest, simple and compound. We refer to interest paid only on the amount originally invested, but not on any interest that accrues subsequently, as simple interest. Simple interest is a function of the principal sum (P) multiplied by rate (r) multiplied by time (n):

$$\text{Interest} = P \times r \times n$$

Example 1

Suppose that Nicola deposits $10,000 in a guaranteed investment certificate (GIC) for a period of one year at 6 percent, the GIC will be worth $10,600 at the end of the year. The interest earned is $600, or 6 percent of the principal amount for one year. If she invested at 6 percent simple interest for four years, the total sum of interest will be $2,400, and the GIC will be worth $12,400 in four years.

In contrast, compound interest refers to the process whereby interest is earned not only on the amount originally invested but also on subsequently accrued interest. Compound interest starts out with exactly the same formula as simple interest but extends it to account for the reinvested interest. Thus, the interest on the second and subsequent time periods is calculated exponentially on the initial principal amount *and* any interest accumulated in preceding time periods.

Example 2

Suppose India invests $10,000 in a GIC at 8 percent compounded annually for two years, the GIC is worth $11,664 at the end of the second year. At the end of the first year the principal and the interest are $10,800 (the same as with simple interest). But the interest for the second year is calculated as 8 percent of $10,800, which is $864

Hence, *future value (FV)* is the amount to which a current Principal (P) will grow at the end of a period of time (n) invested at a compounded rate (r) of interest. The formula is:

$$FV = P(1+r)^n$$

The formula demonstrates that the longer the time horizon for investment and the more frequent the compounding intervals, the greater the future value of a present sum of money.

However, Future Value Tables specify the value of $1 ($P$) at various interest rates (r) over various periods of time (n). The tables are a simple way of applying the formula. Alternatively, one can use a financial calculator.

Example 3

Jasmine invests $5,000 in a TFSA, which will earn 8 percent interest compounded for twenty years. The future value of $1 at 8 percent for 20 years is a factor of **4.66**. Hence, $1 will grow to $4.66 in twenty years. Therefore, $5,000 will grow to $23,300 in that time.

Future value tables typically relate interest rates (*r*) and time (*n*) on an annual basis. Where the compounding is more frequent, we must adjust the *r* and *n* factors to accommodate for more frequent compounding effects.

Example 4

Suppose that in example 6, Jasmine's investment at the 8 percent annual rate is compounded semi-annually instead of annually. Then we adjust *r* to 4 percent (8/2), and *n* to 40 (20*2) semi-annual periods to accommodate semi-annual compounding. The appropriate future value factor of $1 is **4.80**. Hence, $1 will grow to $4.80 and $5,000 will grow to $24,000. The semi-annual compounding yields $700 more than annual compounding.

The more frequent the compounding, the higher the cost of borrowing or the rate of return. We can generalize the future value factor when a contract requires interest compounding more than once a year and still use the tables. When interest is compounded *c* times a year, multiply the number of years that (*P*) will be invested by *c*, and divide the interest rate (*r*) by *c* and then see where the two values intersect in the appropriate table.

Example 5

Suppose that Ravi deposits $2,000 in a tax-free account that pays 6 percent per year, compounded semi-annually, for five years. The bank will pay 3 percent every six months (6%/2) over 10 times periods (5*2). A future value table tells us that 3% intersects with 10 time periods for a factor of **1.34**. Hence, the $2,000 will grow to $2,680 in five years.

The message is simple: a lender will benefit from compounding interest as frequently as possible and the borrower will pay more for frequent compounding. Thus, in an open and competitive market, the borrower and the lender should negotiate *both* the interest rate and the frequency of compounding intervals.

Annuities

An *annuity* refers to a sequence of periodic and equal amounts pursuant to a contractual arrangement. For example, a retiree can purchase an annuity contract from a life insurance company to provide a stream of annual income in retirement. In the simplest case of a single-premium life annuity, the buyer pays a lump sum upfront in return for an annual stream of payments that will last for the buyer's entire life. An annuity is essentially a form of risk transfer from the annuitant to the issuing life insurance company. The value of the annuity will depend upon the amount invested, prevailing interest rates and investment returns.

Forms of Annuities

There are many forms of annuities.

- *Single life annuity*: provides guaranteed income for a lifetime of one individual, regardless of market conditions or interest rate fluctuations.
- *Joint Life Annuity* **(**Last Survivor Annuity): provides guaranteed income for the lives of two people regardless of market conditions or interest rate fluctuations. When the first person dies the income is paid to the surviving individual.

  Annuity Certain: provides guaranteed income for a predetermined period of time or until a certain age.

Annuities may be indexed to inflation and may either be immediate or deferred. Each variation affects the cost and payout ratios.

Example 6 (Single Life Annuity)

70-year old Linda buys an annuity from Life Insurance Company for $1 million today in return for an annual payment of $100,000 for as long as she lives. The annuity is an actuarial bet on how long she will live and the investment return that the Life Insurance Company will earn on the $1 million. If Linda lives to age 90, the insurance company loses the bet because she will collect

$2,000,000. If she dies at age 75, the insurance company wins because it pays out only $500,000. Sophisticated actuaries calculate the odds for insurance companies, which seldom lose their overall bets, in much the same way that casinos in Las Vegas never lose in the long run.

Investors also use annuities in planning for retirement. In an annuity investment, the investor deposits a series of payments into a savings vehicle (tax-sheltered or otherwise) on an annual basis. In an *ordinary annuity* (*annuity in arrears*), the investor makes her payments at the end of the first period. In contrast, under an *annuity due* (*annuity in advance*), the investor makes the payments at the beginning of the period. The future value of both annuities is calculated using the principles of determining future values.

Ordinary Annuity

The future value of an *ordinary annuity* represents the sum accumulated at the *end* of the last payment.

Example 7

Suppose that Jennifer contributes $1,000 at the *end* of each year for a period of four years, which accumulates at 5 percent each year. Then, using Table 5-1:

First payment, will compound for three years (4-1)	1.16
Second payment, will compound for two years (4-2)	1.10
Third payment, will compound for one year (4-3)	1.05
Final payment, no interest	1.00
Total annuity factor	**4.31**

Hence, Jennifer’s annual contribution of $1,000 would accumulate to $4,310 at the end of four years. Alternatively, we obtain the same result in Table 5-4 where 5% and 4 years intersect to provide a factor of 4.31. Table 5-4 essentially uses the future value factors from Table 5-1 to provide a quick answer.

Annuity Due

With an *annuity due* the investor makes payments at the beginning of the period. The future value of an annuity due represents the sum accumulated one period *after* the last payment. Since the last payment is invested for one extra year, the accumulated amount will be greater than that of an equivalent ordinary annuity.

Example 8

Suppose from the previous example, that Jennifer, contributes $1,000 at the *beginning* of each year for a period of four years, which accumulates at 5 percent each year. Then, using Table 5-1:

First payment, will compound for four years	1.21
Second payment, will compound for three years	1.16
Third payment, will compound for two years	1.10
Final payment, will compound for one year	1.05
Total	**4.52**

Jennifer's annuity payment of $1,000 for four years will accumulate to $4,520, which is $210 more than the equivalent ordinary annuity. The reason for the increase is that her last payment of $1,000 has one extra year to earn interest.

Hence, there can be a significant difference in making payments at the beginning of each period (annuity due) rather than waiting to make annual payments at the end of each period (ordinary annuity).

Planning for College

We also use annuities for planning other purposes that will require the use of accumulated capital funds, such as, for example, for education.

Example 9

Frankie's grandparents plan to invest $2,000 at the *beginning* of each year for the next fifteen years in his college fund. Assuming the fund earns 6 percent annual compound interest, how much will the fund have in fifteen years?

Using Table 5-4, because Frankie's grandparents are making fifteen payments to an annuity due, we find that the future value factor for *sixteen* payments is **25.67** for an ordinary annuity due, less1 equals 24.67. Hence, the $2,000 annuity will grow to $49,340 in fifteen years.

Tax-free or Taxable Accounts

We can use annuity values to plan for retirement in tax-free or taxable accounts. Understandably, the amount invested will grow faster in a tax-free account than in a taxable account.

Example 10

Suppose that Sacha (age 35) invests $5,000 at the *end* of each year (ordinary annuity) at 10 percent in a tax-free account, such as an RRSP or TFSA. Table 5-4 provides a factor of **164.49** for 30 years. His tax-free account will grow to $822,450 by the time he is 65 years of age. If, instead, he had a personal tax rate of 40 percent and invested the same amount for 30 years, he would accumulate only $493,470. Hence, rates of return on investments only have meaning after considering the tax bite on earnings. Pre-tax rates of return are meaningless.

Present Value

We evaluate the time value of money in two ways: the future value of a present sum of money or the present value of a future sum. Although this may seem obtuse, both values are in fact merely different ways of looking at the same thing. The future value is the sum to which an amount, or a series of periodic and equal amounts, will grow at the end of a certain amount of time if compounded at a particular interest rate. The present value is the discounted value at a particular rate of interest of a sum of money to be received in the future. Both forms of analysis flow from the fundamental principle that a dollar today is worth more than a dollar tomorrow because one can invest the money today and some interest on it. In other words: *"Time is money"*.

If $1,000 compounded at 10 percent will equal $1,331 in four years, then $1,331 due in four year must be equal to $1,000 today.

Example 11

Year (n)	**Principal (P)**	**Interest rate (r)**	**Interest earned per year**
1	**$1,000**	**10%**	**$100**
2	**$1,100**	**10%**	**$110**
3	**$1,210**	**10%**	**$121**
4	**$1,331**	**10%**	**$133**

We define *present value* as the amount that will grow to a larger sum at the end of "*n*" periods of time in the future, at "*r*" compound rate of interest. The formula simply restates the future value formula to solve for *PV*.

$PV = FV / (1+r)^n$

Where: *PV* is present value

- *FV* is future value
- *r* is the interest rate (as a decimal)
- *n* is the number of years

A present value table shows pre-calculated amounts of the present value of $1 at specified rates of interest (*r*) the end of specified future years (*n*). Present value calculations are always done on the basis of cash flows and not on accrual accounting profits.

Legal Settlements

Lawyers need to determine the time value of money in a variety of circumstances, most notably in litigation, where the issues have to be presented clearly and simply to judges *before* they arrive at their judgements.

Example 12

Suppose that a plaintiff's lawyer is offered a choice between taking an immediate cash settlement of $500,000 or six successive annual payments of $100,000, payable at the end of each year, for a total of $600,000. Should she accept the lump-sum settlement or pursue the extended payment plan? Ignoring questions of risk and insolvency, the financial answer depends upon the prevailing interest rate and income tax considerations associated with the two alternatives. At an interest rate of 10 percent, the factor for six annual payments is **4.35**. The six annual payments of $100,000 have a present value of only $435,300. Hence, the plaintiff would be better off with the nominally larger immediate lump-sum settlement. If the interest (discount) rate was 4 percent, however, the factor for the six payments is **5.24.** Hence, the $100,000 annual payments would have a net present value of $524,000.

We can arrive at the above answer in two ways. We can add the present values of each of the payments for six years at 10 percent from present value table.

Years (*n*)	**Rate (*r*) @ 10%**
1	.909
2	.826
3	.751
4	.683
5	.620
6	.564
Total (rounded)	**4.35**

Hence, the present value of $100,000 payable in six instalments would be $435,300.

Alternatively, we arrive at the same answer by looking at an annuity table. Reading down the 10 percent column and across the six-year row, we see that the present value of an annuity of $1 payable at the end of each year is **$4.35**. Hence, the present value of a $100,000 annuity would be $435,000 (rounded). At 4 percent it would be $524,000.

Similarly, the defendant in a lawsuit involving future lost profits of a business enterprise may ask the court to reduce the size of any lump-sum award to the plaintiff to consider the accelerated value of receiving the money today, rather than over an extended period in the future.

Example 13

Suppose that one can establish that the defendant's actions will cause the plaintiff a loss of $100,000 of business profits annually for a period of five years? Should the defendant be required to pay the nominal amount of the damages ($500,000) up front, or some lesser discounted amount because of the time value of money? At an interest rate of 12 percent, the annual loss of $100,000 spread over five years is worth only $360,000 today if the payments are receivable at the end of each year.

Years (*n*)	**Rate (*r*) @ 12%**
1	.893
2	.797
3	.711
4	.636
5	.567
Total (rounded)	**3.60**

Alternatively, an annuity table provides a factor of 3.60 as the present value of an annuity. Ignoring the time value of money would penalize the defendant and provide the plaintiff with a windfall gain.

Tax Considerations

Time value considerations can also raise income tax considerations depending on the characterization of the terms of settlement or judgement. Where the lump sum amount is a substitution of a taxable stream of earnings, the discounted value would be on similar account. Where the amount is on account of a non-taxable capital settlement, the lump sum would not normally be taxable if we followed the substitution theory. There can, however, be other considerations if the amount that would otherwise be taxable is paid as lump sum upon death. Thus, time value considerations should be considered in structuring damage settlements to account for the net present value of money *and* the related tax costs and savings.

The more distant the time when an amount has to be paid on account of taxes, the lower its present value. Hence, a fundamental principle is to attempt to legally delay or defer the payment of taxes as long as possible so as to minimize the present value of the obligation today. This basic principle of finance is at the root of most legitimate tax shelter schemes. Unfortunately, the principle is also used to promote dubious tax shelter arrangements, such as, charitable donation schemes, with unfortunate consequences for investors.

Example 14

For example, the present value of $1 invested at 8 percent at the end of years 1 through 5 is as follows:

Years (*n*)	**Rate (*r*) @ 8%**
1	.93
2	.86
3	.79
4	.74
5	.68

Hence, when we say that the interest rate is 8 percent per year, we mean that we should be able to exchange 93 cents today in return for $1 a year from now, and 86 cents in return for $1 two years from now, and so on. If we can defer the obligation to pay $1,000 in taxes today for five years, the present value of the liability to pay is only $680. Although the face amount of the legal obligation to pay remains the same, namely, $1,000 in five years, its economic value varies depending upon when it is paid. Stated another way, if we invested $681 in a deposit that compounded tax free at a rate of 8 percent, we would accumulate $1,000 at the end of five years with which to pay the tax liability. By deferring the tax payable for five years we reduce the tax liability by $320.

Example 15

To continue with the same example above, assume that we can defer tax payable for a period of fifty years. The present value of $1,000 payable in fifty years at a discount rate of 8 percent is $21. In other words, if we invest $21 today and compound it for fifty years *net* 8 percent annually, we will have $1,000 at the end of the investment period. The discounted amount shrinks as a function of two factors: time (n) and the discount rate (r). The longer the period of deferral and the higher the interest rate, the lower the discounted present value. Hence, $1,000 payable in fifty years and discounted at 20 percent has a present value today of (effectively) zero.

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