TIME VALUE OF MONEY

By

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“Time is money”.

Money has no monetary value except when it is put in the context of time. Stated alternatively: “A dollar today is worth more than a dollar tomorrow”. The value is the effect of time on invested money. This is the most basic principle of finance.

Interest is the rental cost of borrowing money. For the borrower, interest is the rent paid for borrowing money; for the lender the income earned from lending money. They are just different sides of the same coin. The concept of the time value of money is relevant in finance, accounting, law, and taxation.

Implications for Investors

There are two dimensions to value: future and present. Given a sum of money, we can determine its value at some future date if we know the interest rate at which we can invest the money. Conversely, if we know that we are to receive a sum of money in the future, we can determine its value today if we know the rate at which it is, or can be, invested. Thus, all money has “time value.”

There are two factors to consider in determining the present or future value of a sum of money: (1) the prevailing interest rate; and (2) the tax consequences of the two options. In lawsuits, it is the responsibility of counsel to raise time value issues to determine the discounted value of judgements looking to lost past profits or anticipated future earnings. Judges will not make such calculations on their own initiative.[1]

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[1] Lehrman v. Gulf Oil Corp. , 500 F. 2d 659, 672 (5th Cir. 1974, *cert. denied*, 420 U.S. 929 (1975).

The value of money depends upon three factors:

1. Principal amount (*P*);
2. Interest rate (*r*); and
3. Time (*n*).

“Principal” is the amount of money originally borrowed or loaned. “Interest” is compensation for use of money [similar to rent]. Time refers to the period of borrowing, lending or investing.

All assets, tangible and intangible, can be expressed in terms of their future or present value if we can determine the rate at which the principal amount of the asset is invested or discounted over a period of time.

The appropriate rate is usually the market-determined interest or investment rate.
In economic terms, “interest” is the rental cost of borrowing money. As with all rentals, the cost of renting money may be fixed in advance, determinable at a future time, or variable according to specified conditions. Thus, interest and time are inextricably related. An interest rate is relevant if, and only if, it is specified in relation to time.

Although most of us are familiar with the calculation of simple and compound interest, we are less intuitive about the concept of the discounted value of future sums of money. This is because we are taught to think intuitively of investing for the future but not of the present value of sums of money to be received in the future. Yet, mathematically speaking, future value and present value are mirror images of each other looked at from different perspectives, such as the opposite ends of a telescope. The primary purpose of determining future and present values is to measure money in comparable terms across time periods by translating future dollars into economically equivalent current dollars, and *vice versa*.

Implications for Lawyers

Lawyers deal with assets that have a "time value" in virtually all aspects of commercial practice in a variety of transactions. The following are some common areas where the principles of time value will arise:

- Matrimonial settlements;
- Computation of damages in litigation;
- Valuation of charitable donations;
- Valuation of life and remainder interests of trusts;
- Truth in lending laws [example: interest on credit cards];
    - Usury laws under the *Criminal Code*;
- Funding for children's education;
- Use of annuities in life insurance products;
- Retirement planning;
- Compounding interest on outstanding tax assessments*;*
- Tax deferral and tax planning.

In addition to the above, all valuation decisions involving business and estate planning involve the time value of money. Although the context of usage may vary, the underlying principles are always the same: *"Time is money"*.

Financial Implications

The time value of money is intrinsic to financial and investment decisions, such as the valuation of equities and bonds. There are two underlying concepts to the time value of money: simple interest and compound interest.

Simple Interest

We start with the concept of interest and simple arithmetic.

Example 1

Suppose that Andy has $1,000, which he can invest at 10 percent simple interest per year.

If he invests the $1,000 for one year, he will earn $100 at the end of the year. If he invests for the second year, his interest will be determined on the *original* principal amount only ($1,000) but without any interest added, according to the same formula. He will continue to earn the same interest ($100) each year according the formula.

Year (n)	Principal (P)	Interest rate (r)	Interest earned
1	$1,000	10%	$100
2	$1,000	10%	$100
3	$1,000	10%	$100
4	$1,000	10%	$100

The formula is:

Interest = P x r x n

Where P = the principal amount invested;

r = the rate of interest for the time period; and

n = the time period.

The "n" in the formula is always equal to 1. At the end of four years, he will have earned $400, which equals 40 percent on his original principal of $1,000.

Compound Interest

When we compound interest, the interest earned each year is calculated on the principal amount (*P*) *and* the interest earned in the preceding year.

Example 2

Suppose that India invests $1,000 for four years compounded at 10 percent per year. Then, at the end year 1, she will have earned $100 (same as simple interest), which is then added to the principal amount at the beginning of year 2. Hence, P in year 2 now equals $1,100. Then, she earns 10 percent on the enhanced principal of $1,100 and will receive $110. And so on. At the end of four years, she will have earned $464, which is 46 percent on her original investment of $1,000.

Year (n)	Principal (P)	Interest rate (r)	Interest earned per year
1	$1,000	10%	$100
2	$1,100	10%	$110
3	$1,210	10%	$121
4	$1,331	10%	$133

The compounding effect increases her rate of return by 6 percent over four years.

Example 3

Suppose that Milo invests $10,000 for four years compounded at 10 percent per year. Then, at the end of year 1, he will have earned $1000 (same as simple interest), which is then added to the principal amount at the beginning of year 2. Then, he earns 10 percent on the enhanced principal of $11,000 and will receive $1100 in year 2. And so on. At the end of four years Milo would have earned $4,640 (rounded), which is 46 percent on his original investment of $10,000.

Note, despite their different absolute earnings, the ratio of India's and Milo's earnings remains the same at 1.46. In other words, regardless of the principal amount, the ratio for 10 percent for 4 years remains the same. Someone has pre-calculated the numbers in the body of Future Value Tables for different Rates (r) and Times (n), so that each ratio represents the shortcut for a specified combination of Rate and Time.

We arrive at the same result by applying a formula to determine future compounded values:

$FV = P(1+r)^n$

Where FV = future value

P = the principal amount invested;

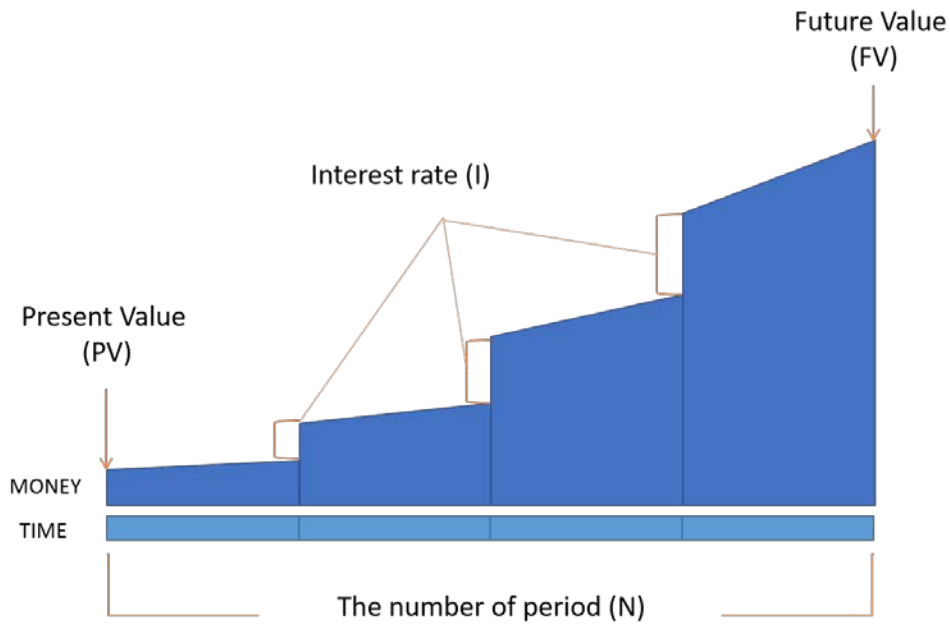
r = the rate of interest for the time period; and

n = the time period.

However, the formula in this case is exponential in that the variable (n) is now the power, rather than the base. Applying this exponential function, we obtain:

$FV = 1,000(1+.1)^4$

FV = $1,460



The Rule of 72

The Rule of 72 allows us to calculate the approximate length of time or interest rate that it will take to double an investment using compound interest. For example, assuming a compound interest rate of 8 percent, it will take approximately nine years (72/8) to double an investment. We can estimate the rate of interest required to double money in a specified number of years. For example, given an interest rate of 6 percent, it will take twelve years (72/6) to double an investment.

The Rule of 72 illustrates the power of compounding interest in planning for retirement, paying off debt, and investing for the long run. Albert Einstein (*Time's* "Person of the Century") is reported to have said that compound interest is "the greatest invention in human history," "the most powerful force in the universe," and the eighth wonder of the world."

Retirement Planning

How much money will an individual require on retirement? Assume that Sacha invests $30,000 at 8 percent (average long-term rate of return on equities) in a tax-free account when he is 25 years old.

Age (years)	**Investment @ 8 percent**
25	$30,000
34	$60,000
43	$120,000
52	$240,000
61	$480,000
70	$960,000

Without ever investing another dollar, Sacha will accumulate $960,000 by the time he is 70.

Berkshire Hathaway Inc.'s performance illustrates the power of compounding. For the period 1965-2019, Berkshire's compounded annual gain equalled 20.3% compared to the S&P 500 (dividends included) gain of 10.0%. The overall gain was 2,744,062% (See: Berkshire's 2019 Report to Shareholders).

Compounding Long-term Debt

We see the dramatic impact of compounding in a recently reported incident in the sleepy hamlet of Mittenwalde in Eastern Germany. According to Reuters, the town historian (Vera Schmidt) uncovered a centuries-old debt slip dating back to 1562 in the archives where it had been filed. Mittenwalde apparently lent Berlin 400 Guilders on 28 May 1562 which was to be repaid with 6 percent interest per year. According to Radio Berlin Brandenburg, adjusting the debt for compound interest and inflation, the total debt now lies in the trillions. Apparently, the Mayor of Mittenwalde and his predecessors have asked Berlin for the return of their money and have made such a request every fifty-five years since 1820, but always to no avail. Mittenwalde, which had a population in 2012 of 8,800 people, would benefit enormously from repayment. Berlin, however, is not playing along. Why?

Future Value

As noted above, there are two forms of interest, simple and compound. We refer to interest paid only on the amount originally invested, but not on any interest that accrues subsequently, as simple interest. Simple interest is a function of the principal sum (P) multiplied by rate (r) multiplied by time (n):

$$\text{Interest} = \text{P} \times r \times n$$

Example 4

If Nicola deposits $10,000 in a guaranteed investment certificate (GIC) for a period of one year at 6 percent, the GIC will be worth $10,600 at the end of the year. The interest earned is $600, or 6 percent of the principal amount for one year. If she invested at 6 percent simple interest for four years, the total sum of interest will be $2,400, and the GIC will equal $12,400.

In contrast, compound interest refers to the process whereby interest is earned not only on the amount originally invested but also on subsequently accrued interest. Compound interest starts out with exactly the same formula as simple interest but extends it to account for the reinvested interest. Thus, the interest on the second and subsequent time periods is calculated exponentially on the initial principal amount and any interest accumulated in preceding time periods.

Example 5

Suppose one invests $10,000 in a GIC at 8 percent compounded annually for two years, the GIC is worth $11,664 at the end of the second year. At the end of the first year the principal and the interest are $10,800 (the same as with simple interest). But the interest for the second year is calculated as 8 percent of $10,800, which is $864

Future value is the sum to which an amount, or a series of periodic equal amounts, will grow at the end of a period of time, invested at a particular compound interest rate. Hence, *future value (FV)* is the amount to which a current Principal (*P*) will grow at the end of a period of time (*n*) invested at a compounded rate (*r*) of interest:

$$FV = P(1+r)^n$$

Future Value Tables specify the value of $1(*P*) at various interest rates (*r*) over various periods of time (*n*). The tables are a simple way of applying the formula. Alternatively, one can use a financial calculator.

Example 4

Jasmine invests $5,000 in a TFSA, which will earn 8 percent interest compounded for twenty years. The future value of $1 at 8 percent for 20 years is a factor of **4.66**. Hence, $1 will grow to $4.66 in twenty years. Therefore, $5,000 will grow to $23,300 in that time.

Future value tables typically relate interest rates (*r*) and time (*n*) on an annual basis. Where the compounding is more frequent, we must adjust the *r* and *n* factors to accommodate more frequent compounding effects. The formula demonstrates that the longer the time horizon for investment and the more frequent the compounding intervals, the greater the future value of a present sum of money.

Example 5

Suppose that in example 4 the 8 percent annual rate is compounded semi-annually. Then we adjust *r* to 4 percent and *n* to 40 semi-annual periods to accommodate semi-annual compounding. The appropriate factor of $1 is **4.80**. Hence, $1 will grow to $4.80 and $5,000 will grow to $24,000. The semi-annual compounding yields $700 more than annual compounding.

The more frequent the compounding, the higher the cost of borrowing or the rate of return. We can generalize the future value factor when a contract requires interest compounding more than once a year and still use Table 5-1. When interest is compounded *c* times a year, multiply the number of years that *P* will be invested times *c* and divide the interest rate (*r*) by *c* and then see where the two lines intersect on Table 5-1.

Example 6

Suppose that Ravi deposits $2,000 in a tax-free account that pays 6 percent per year, compounded semi-annually, for five years. The bank will pay 3 percent every six months (6%/2) over 10 times periods (5*2). A future value table gives us a factor of **1.34**. Hence, the $2,000 will grow to $2,680 in five years.

The lesson is simple: a lender will benefit from compounding interest as frequently as possible and the borrower will pay more for frequent compounding. Thus, in an open and competitive market, the borrower and the lender must negotiate *both* the interest rate and the frequency of compounding intervals.

Impact of Tax Rates

There is no negotiation on interest rates in tax law. The *Income Tax Act* and *Regulation 4301* prescribe the rates to be charged and they are set each quarter. There are two principal rates:

1. The rate of interest payable by a taxpayer to the CRA on late taxes;
2. The rate of interest payable by the CRA to a taxpayer on refunds.

The prescribed rates are as follows:

	Refunds (%)	Late Taxes (%)
2020 Q1	4	6

The rate of interest paid to taxpayers is taxable as income. The rate payable on late taxes is non-deductible and must be paid with after-tax dollars. Translated, this means that a taxpayer with a 50 percent marginal tax rate will need to earn 12 percent income to pay 6 percent in late taxes.

CRA compounds interest on a *daily* basis on outstanding amounts of taxes payable [§ 161(1) and §248(11) *ITA*]. Hence, it is almost always to a taxpayer's advantage to pay his or her taxes on a contentious assessment and then challenge its validity at a later date. It is virtually impossible for a taxpayer to obtain an alternative investment that will yield an equivalent amount of interest for the same risk. Since tax disputes can continue for 10 to 15 years, the daily compounding effect has an enormous financial impact on the ultimate amount payable if the taxpayer ultimately loses the appeal.

Example 7

Suppose that an individual owes $50,000 in assessed taxes. If unpaid, at a rate of 6 percent (Q1, 2020), the tax bill will climb to $91,100 in 10 years. The $41,000 of interest payable will not be deductible for tax purposes. Hence, a taxpayer with a 50 percent marginal tax rate would need to earn $82,000 just to pay the interest.

If the taxpayer pays the $50,000 assessment upfront and wins his case after 10 years, he will receive his $50,000 together with interest of $24,590, which will be fully taxable. Hence, a 50 percent marginal rate taxpayer would retain only $12,295 net of taxes.

Impact of Tax Rates on Future Values

Taxes play a significant role in determining future values. Taxes payable reduce the amount of interest available for reinvestment (the "*r*" in the formula) and, hence, reduce the compounding effect on the principal sum.

Example 8

Suppose that Nicola invests $10,000 in a tax-sheltered investment that earns 10 percent compounded annually for ten years. At the end of ten years, the future value of the investment will be $25,900. The accumulated gain of $15,900 represents the *gross* compound interest over a period of ten years. Thus, the money more than doubles in ten years.

Example 9

If Nicola pays tax at 40 percent on her earned interest on a current basis, she will have only 6 percent to reinvest each year. The future value of $10,000 invested at 6 percent (net) will be $17,900 at the end of ten years. Thus, the ultimate value of the net after-tax investment is $8,000 (31 percent) less than its value in a tax-sheltered investment.

Here we see the simple mathematics of tax planning for investments, retirement, tax shelter programs, and the benefits of tax deferral. Since taxes decrease the amount that can be reinvested, it generally pays to defer the payment of taxes (the longer, the better) provided that the CRA is not levying interest on the outstanding amount on a current basis.

Example 10

A $1,000 investment compounding annually at 20 percent in a tax shelter is worth $1.47 million after forty years. If the investment is taxed annually at 25 percent, the net return is 15 percent and the investment is worth approximately $267,000 in forty years.

If taxed at 40 percent, the net return is 12 percent and the investment is worth only $93,100 after forty years. Taxes erode investment returns. Hence, tax deferral is one of the most effective means of retirement and estate planning.

The Rule of 72 also illustrates the impact of taxes on investments. We saw earlier that an investment at 6 percent will double in twelve years. Each $1 would be worth $2.01 at the end of 12 years. If taxes reduce the reinvestment rate by 40 percent to 3.6 percent, it will take 20 years for the money to double. Thus, reinvestment gross of tax is always preferable to reinvestment net of tax. Compounding for long periods, even at modest rates, leads to explosive growth. At 3 percent, for example, $1 will grow to $6,874 billion in 1,000 years!

Nominal and Effective Interest Rates

When we speak of interest rates, we must distinguish between "nominal" and "effective" rates. The difference between the two depends upon the method of calculation and the frequency of the compounding period. For example, when we say that a person has a bank loan at 10 percent annual rate of interest, that is the nominal rate. The effective rate of interest will also be 10 percent, but only if interest is calculated at year-end. As we saw above, if the compounding period is more frequent, the effective rate of interest increases. This is important with credit cards that disclose an annual percentage rate (APR). If the APR is 18 percent annually, the monthly charge rate is 1.5 percent, which makes the effective rate of interest 19.56 percent.

Section 347 of the *Criminal Code* of Canada makes it a criminal offence to charge usurious interest rates, which is defined as annual interest that exceeds 60 percent. The purpose of the provision is to discourage loan sharking. In fact, the provision is rarely applied against "loan sharks", who, instead, use unorthodox collection methods. The provision is more often used against legitimate commercial lenders.

In *Garland v Consumers' Gas Co*, for example, the defendant gas company sold its gas to consumers under a sales agreement that contained a late penalty payment (LPP) clause that required customers to pay a charge of 5 percent on monthly unpaid bills if payment was not made within sixteen days. On an annual basis, the LPP violated Section 347 of the *Criminal Code* if a customer paid within thirty-seven days, but not if a customer paid after that period. Applying a "wait-and-see" approach, which determines the effective rate of interest when payment is *actually* made, the Supreme Court of Canada held that the LPPs violated the *Criminal Code*.

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